Augmented Lagrangian

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Solves the following constrained optimization problem:

$\min_x f(x)$
s.t. $c_I(x) \leq 0, c_\epsilon(x) = 0$

Unconstrained Optimization Problem

$L_A(x, \lambda, \mu) = f(x) + \lambda^\top c(x) + \frac{1}{2} c(x)^\top I_\mu c(x)$
where

$$I_{\mu,ii} = \begin{cases} 0 & \text{if } c_i(x) < 0 \wedge \lambda_i = 0, \text{ for } i \in \mathcal{I} \\ \mu_i & \text{otherwise.} \end{cases}$$

Let us interpret this weird matrix I_μ.

<u>Equality Constraints:</u> $c_E(x) = 0$, always enforced, meaning that $I_{\mu,ii} = \mu_i$
<u>Inactive Inequality Constraints:</u> $c_i(x) < 0$ and $\lambda_i = 0$, meaning that $I_{\mu,ii} = 0$
<u>Active Inequality Constraints:</u> $c_i(x) > 0$ and $\lambda_i > 0$, meaning that $I_{\mu,ii} = \mu_i$

Basically, the quadratic term is always on except for inactive inequality constraints.

Dual Update

We solve using any type of convex solver:
$\hat{x}^* \approx \arg\min_x L_A(x, \lambda, \mu)$
What is the gradient of the unconstrained optimization problem?
$\frac{\partial L_A}{\partial \lambda} = c(x)$

Now we want to increase the constraints:
<u>Equality:</u> $\lambda_{i+1} = \lambda_i + \mu_i c_i(\hat{x}^*)$
<u>Inequality:</u> $\lambda_{i+1} = \max(0, \lambda_i + \mu_i c_i(\hat{x}^*))$

We increase the penality monotonically with a schedule $\phi_i > 1$
$\mu_{i+1} = \phi_i \mu_i$