# Line Search

Thursday, 5 February 2026 21:39

## Background Information

**Iterative LQR**
Start with a guess for $X, U$ (random noise, Bdot)

Evaluate $J = l_f(x_N) + \sum_{k=0}^{N-1} l(x_k, u_k)$
while $|J - J_{prev}| > tolerance$
$J_{prev} = J$
$K, d, \Delta V \leftarrow \boldsymbol{BackwardPass}(X, U)$
$X, U, J \leftarrow \boldsymbol{ForwardPass}(X, U, K, d, \Delta V, J_{prev})$
$return\ X, U, J$

**Backward Pass**

$$p_N = \frac{\partial l_f}{\partial x}, P_N = \frac{\partial^2 l_f}{\partial x^2}$$

for $k = N-1:-1:0$ (Bellman's Optimality Principle)

$$\delta Q_k = \frac{1}{2}\begin{bmatrix}\delta x_k \\ \delta u_k\end{bmatrix}^\top \begin{bmatrix} \frac{\partial^2 l}{\partial x_k^2} + A_k^\top P_{k+1} A_k + \frac{\partial c_k}{\partial x_k}^\top I_{\mu,k} \frac{\partial c_k}{\partial x_k} & \frac{\partial^2 l}{\partial x_k \partial u_k} + A_k^\top P_{k+1} B_k + \frac{\partial c_k}{\partial x_k}^\top I_{\mu,k} \frac{\partial c_k}{\partial u_k} \\ \frac{\partial^2 l}{\partial u_k \partial x_k} + B_k^\top P_{k+1} A_k + \frac{\partial c_k}{\partial u_k}^\top I_{\mu,k} \frac{\partial c_k}{\partial x_k} & \frac{\partial^2 l}{\partial u_k^2} + B_k^\top P_{k+1} B_k + \frac{\partial c_k}{\partial u_k}^\top I_{\mu,k} \frac{\partial c_k}{\partial u_k} \end{bmatrix} \begin{bmatrix}\delta x_k \\ \delta u_k\end{bmatrix} + \begin{bmatrix} \frac{\partial l}{\partial x_k} + A_k^\top p_{k+1} + \lambda_k \frac{\partial c_k}{\partial x_k} + \frac{\partial c_k}{\partial x_k} I_{\mu,k} c_k \\ \frac{\partial l}{\partial u_k} + B_k^\top p_{k+1} + \lambda_k \frac{\partial c_k}{\partial u_k} + \frac{\partial c_k}{\partial u} I_{\mu,k} c_k \end{bmatrix}^\top \begin{bmatrix}\delta x_k \\ \delta u_k\end{bmatrix}$$

if $Q_{uu}$ invertible:
$\delta u_k = -(Q_{uu} + \rho I)^{-1}(Q_{ux}\delta x_k + Q_u)$
else:
$\rho \uparrow$ and try again

return sequence of $K, d, \Delta V$ for every timestep

**Forward Pass**

$\alpha = 1$
$for\ k = 0:1:N-1$
$\bar{u}_k = u_k + K_k(\bar{x}_k - x_k) + \alpha d_k$
$\bar{x}_{k+1} = f(\bar{x}_k, \bar{u}_k)$

$$J = l_f(x_N) + \sum_{k=0}^{N-1} l(\bar{x}_k, \bar{u}_k)$$

If $J$ has actually decreased (line search condition), return. Else, decrease $\alpha$ and try again

### Choosing $\alpha$

Choosing $\alpha$, or how aggressively we want to track a feedback controller, is tricky business. The main performance criterium is an adequate decrease in cost.

We may determine a radio of actual decrease to expected decrease:

$$z = \frac{J(X, U) - J(\bar{X}, \bar{U})}{-\Delta V(\alpha)}$$

Recall from earlier that $\Delta V_k = Q_u^\top d_k + \frac{1}{2} d_k^\top Q_{uu} d_k$, except now we want all timesteps:

$$\Delta V(\alpha) = \sum_{k=0}^{N-1} \alpha d_k^\top Q_u + \alpha^2 \frac{1}{2} d_k^\top Q_{uu} d_k$$

As you may notice, since we are running our controller with $\alpha d_k$, our expected cost should also use $\alpha d_k$.

#### Failure in $\alpha$

We accept a trajectory if $z \in [0.0001, 10]$. If not, we decrease scaling by $\alpha \leftarrow 0.5\alpha$.

#### Failure in $\rho$

If the **Forward Pass** still fails repeatedly, go to **Backward Pass** and increase the regularization.