Projected Newton Method

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The AL-iLQR slows down significantly after the constraints get large. The solution is likely already quite good, but especially active constraints may not be exactly fulfilled.

Projected Newton Method

Take the primal and dual trajectories from the AL-iLQR method $Y \leftarrow X, U, \lambda$ and use them to warm-start an active-set projected Newton method.

Active Set: Constraint violation above some small $\epsilon_{constraint}$.

$$\min_{\delta z} \delta z^\top H \delta z$$
$$s.t. D\delta z = d$$

Where δz is the stacked states and controls at all timesteps, and D are the active linearized constraints.

Steps toward manifold

The proper update step (solution of quadratic) is:
$$\Delta Y = -H^{-1}D^\top (DH^{-1}D^\top)^{-1} d$$

Where d is the current violation and $D = \frac{\partial c}{\partial Y}$ is the Jacobian of the constraints.
We can make it more numerically stable by doing $S = \sqrt{DH^{-1}D^\top}$
$$\Delta Y = H^{-1}D^\top (S^{-1}S^\top d)$$

Finally, at the end we calculate the maximum residual $v = \left|\left|d\right|\right|_\infty$

Minimization Representation

The above steps are the closed-form solution to:
$$\Delta Y = \arg\min_{\Delta Y} \frac{1}{2} \Delta Y^\top H \Delta Y$$
$$s.t. c(Y) + D\Delta Y = 0$$